Lecture: 5-5 The Substitution Rule (Part 1)

Example 1: How would we factor $x^4 - 5x^2 + 6$ and how might it relate to finding $\int 2x\sqrt{1+x^2}dx$?

The Substitution Rule If $u = g(x)$ is a differentiable function whose range is an interval I and f is continuous on I then

$$\int f(g(x))g'(x)dx = \int f(u)du$$

Note, the substitution rule is basically undoing the ______________ rule.

Example 2: Evaluate $\int x^3 \cos(x^4 + 2)dx$ two different ways:

(a) solve for dx.

(b) solve for $x^3 dx$.

The trickiest thing about subsitution is deciding what to substitute. As substiution is (usually) undoing the chain rule you chould let your u be the inside function. Choose u to be the stuff inside of a power, root sign, denominator, or trigonometric function. When you are choosing your u the derivative of u should appear elsewhere in the integrand up to a constant multiple. The only way to get better is a lot of practice!
Once you make your subsitution the integral usually simplifies considerably. If your original variable does not completely disappear when making the subsitution you either (a) chose a subsitution that doesn't work or (b) made a mistake. At this stage you can try something different, or start your original substitution again.

Example 3: Evaluate the following indefinite integrals.

(a) $\int \sqrt{3x+2}dx$

(b) $\int \cos^4 x \sin x dx$

Example 4: Evaluate the following indefinite integrals.

(a) $\int \frac{\sec^2 x}{\tan^2 x}dx$

(b) $\int \frac{x}{\sqrt{1-x^4}}$

Example 5: Evaluate the following indefinite integrals.

(a) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx$

(b) $\int \frac{\arctan x}{x^2+1}dx$